MAKING RATIO MEASUREMENTS – THE RIGHT WAY

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DL INSTRUMENTS, LLC
233 Cecil A. Malone Dr., Ithaca, New York 14850; Phone 607-277-8498; Fax 607-277-8499

I INTRODUCTION

Ratiometric techniques compensate for fluctuations in signal drive to a measurement apparatus. Examples of this would include photometry in which the light source varies or impedance bridges in which the excitation level changes with frequency. By measuring the signal ahead of the experiment and dividing its value Y into the experimental result X, the actual relative effect of the experiment can be determined. The ratio R is generally normalized to unity by introducing a normalizing factor K for some nominal or initial values of X and Y.

$$R = K\frac{X}{Y}$$

For applications in which high levels of interference tend to obscure an ac measurement, the lock–in amplifier is often employed to recover the signal. A variety of ratiometric methods exist for use with lock–ins, ranging from the simple and relatively inexpensive to more elaborate and costly solutions. The purpose of this document is to guide the user in selecting the setup which will yield adequate performance within the constraints of his budget. The examples given, in all cases, are taken from the field of optical measurements, since it is in this area that most ratio requirements arise. In these examples a mechanical chopper or pulsed laser modulates the light source, while the lock–in serves to demodulate the electro–optical ac output of the detector to an observable dc level, X. This signal is then divided by a second dc input Y, either by purely analog means or by first digitizing X and Y then computing R. The digital method is potentially much more accurate.

II DC DENOMINATOR TECHNIQUES

The setup shown in Figure 1 will suffice to cancel changes in light intensity due to lamp intensity drift and due to monochromator wavelength changes. It will not, however, be effective against stray room light falling on the Y detector nor against background radiation, due to lack of ac modulation in the Y path. It is also susceptable to errors due to differential detector response as a function of wavelength or temperature. Furthermore, if the incandescent lamp and monochromator were replaced by an unstable

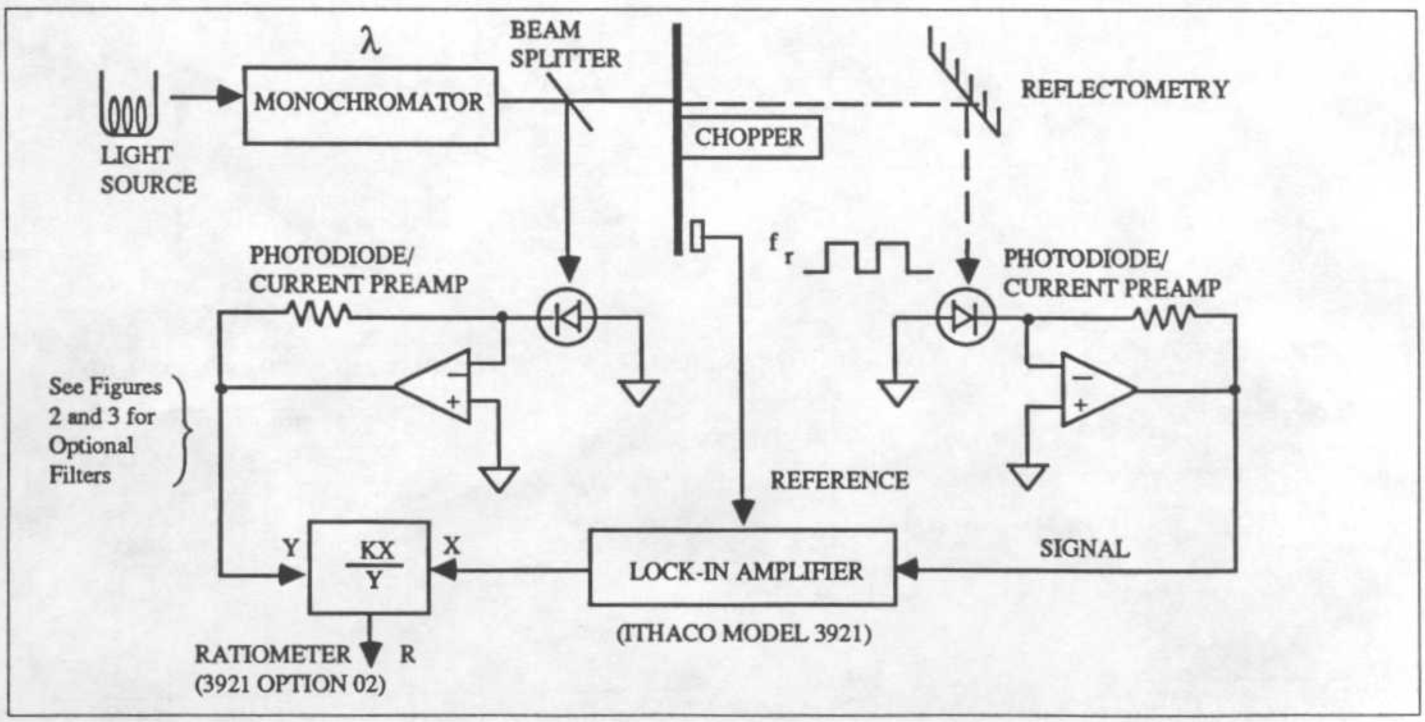


FIGURE 1 A SIMPLE TWO DETECTOR RATIO SETUP USING DC DENOMINATOR TECHNIQUE

source, such as a laser exhibiting mode jumping, it would be unable to dynamically cancel rapid source fluctuations. If the ratiometer is an analog type as shown, it will yield approximately 1% to 2% linearity over a 100 mV to 10 V dc Y input range, regardless of the quality of the lock–in amplifier. These considerations illustrate the limitations of a simple low cost approach to ratio measurements. This is not to say that Figure 1 represents a useless arrangement: it would be perfectly appropriate in some situations. Notice that a dual detector arrangement as shown would be employed if the signal detector must be movable to accommodate various angles of incidence.

The setup shown in Figure 2 overcomes the problems of relative detector response by combining the numerator and denominator beams onto a single detector. This would be possible assuming the detector can be mounted in a fixed location and that the split ratio can be set to achieve a large value of Y (e.g., 1 volt dc) compared to X (e.g., 1 mV rms) at the output of the preamplifier. Note that the lock–in responds only to the ac portion of the preamp output. Also note the small error component in the Y value due to the superposition of the average X value. As in Figure 1, the detector in Figure 2 is susceptable to ambient radiation.

The RC filter in the Y path can be inserted to reduce ripple in the ratio output. It can also be used for another important purpose; dynamic cancellation of short term laser output level changes. This is described mathematically in Appendix A. By exactly matching the electrical transfer functions introduced in the X path by the lock–in and in the Y path by the RC filter, accurate ratio measurements can be obtained even for quite large steps in source output. For this to work, t = RC must be set to match the lock–in time constant T. Furthermore, for the RC filter shown in Figure 2, the lock–in must be set to the 6 dB/octave rolloff mode. Figure 3 depicts the RC filter required to match a 12 dB/octave lock–in time constant rolloff. These one or two section filters can be inserted as required in the Y path for setups similar to Figures 1, 2 and 4 to achieve dynamic ratio capability.

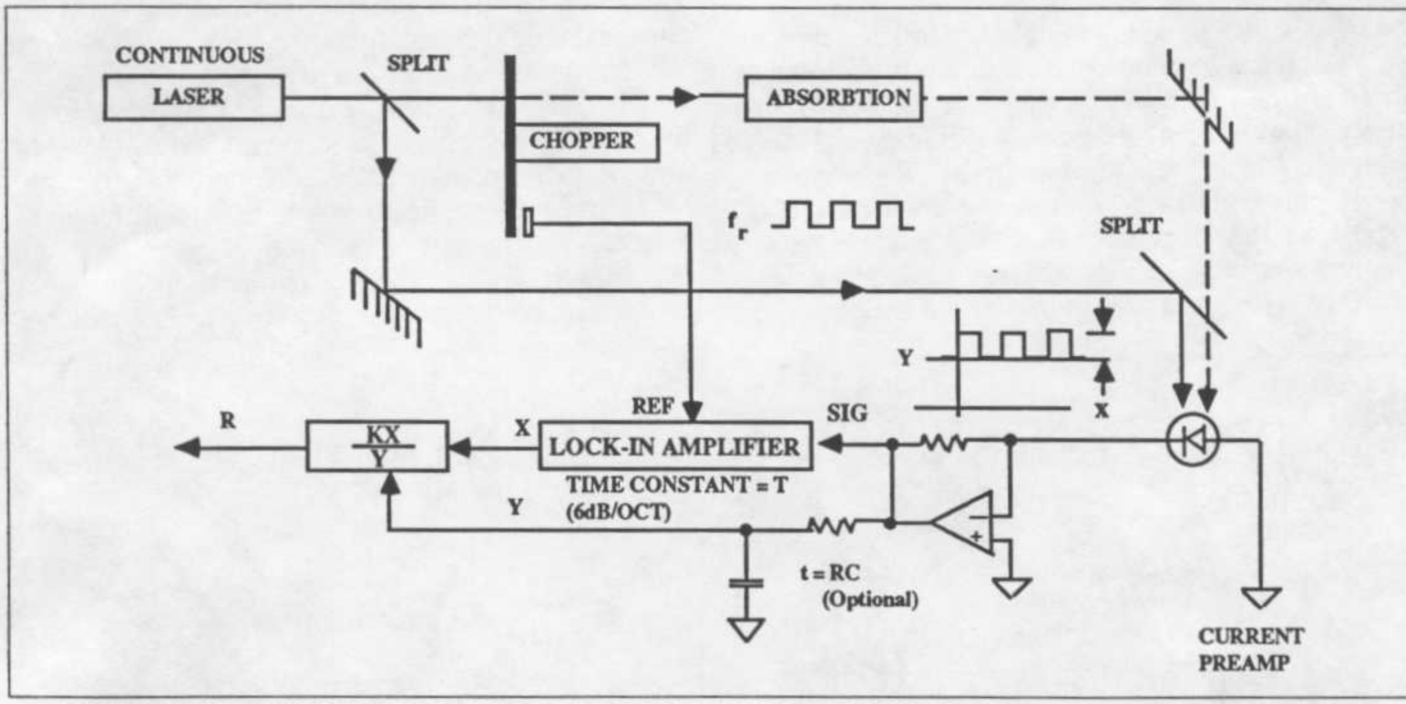


FIGURE 2 AN EVEN SIMPLER SINGLE DETECTOR RATIOMETER USING DC DENOMINATOR TECHN IQUE

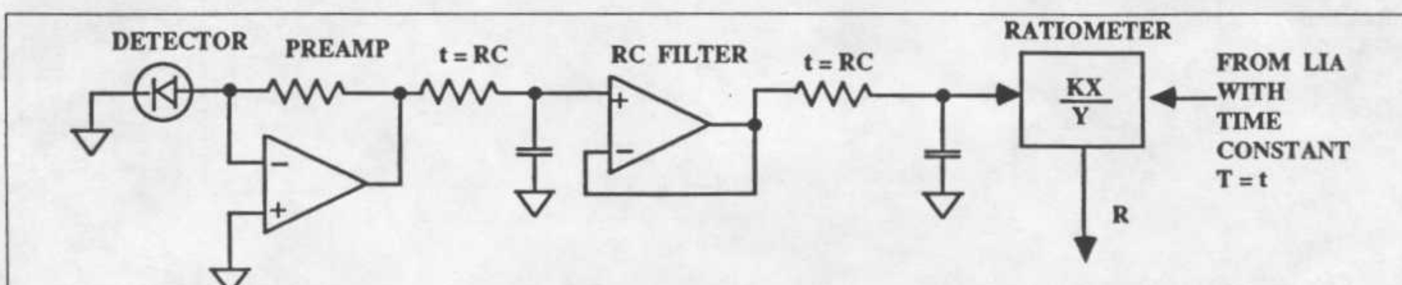


FIGURE 3 DENOMINATOR RC FILTER FOR 12 dB/OCTAVE LOCK–IN TO ACHIEVE DYNAMIC RATIO PERFORMANCE.

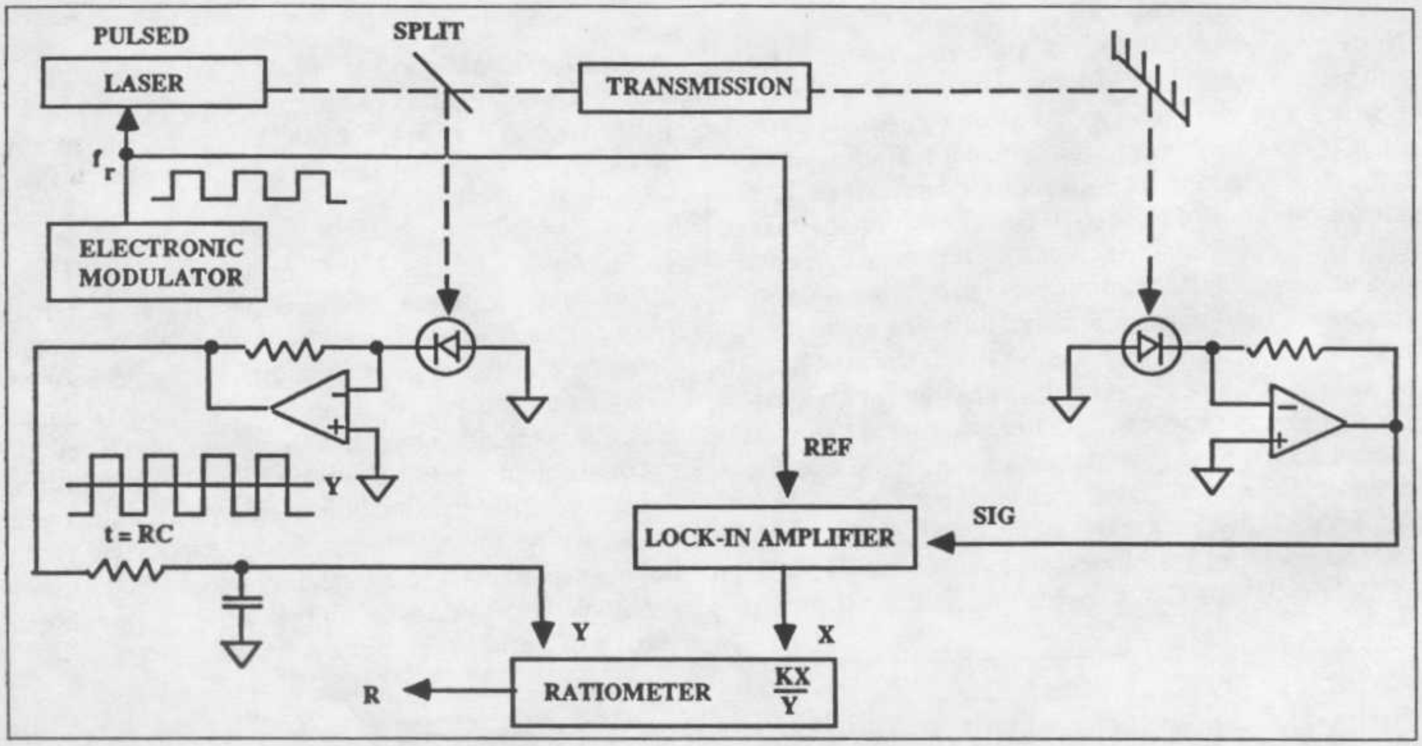


FIGURE 4 DUAL DETECTOR RATIOMETRY, MODULATED DENOMINATOR, SINGLE LOCK–IN

The setup in Figure 4 presents another variation on the dc denominator theme. Here the pulse modulated laser offers no opportunity for getting a dc Y signal directly, so one is obtained by averaging the denominator pulse signal. The t = RC filter value is chosen sufficiently long to smooth the Y signal to the desired degree, and need not necessarily match the lock–in time constant setting. Figure 4 offers no performance improvement over that shown in Figure 1. We are still at risk from ambient light reaching the Y detector.

The arrangement shown in Figure 4 will not be effective in dynamically cancelling laser pulse–to–pulse jitter with respect to width or position. It will work for pulse height jitter, assuming the RC filter matches the lock–in time constant and rolloff settings. The reasons for this will be brought out in Section V, which discusses phase effects.

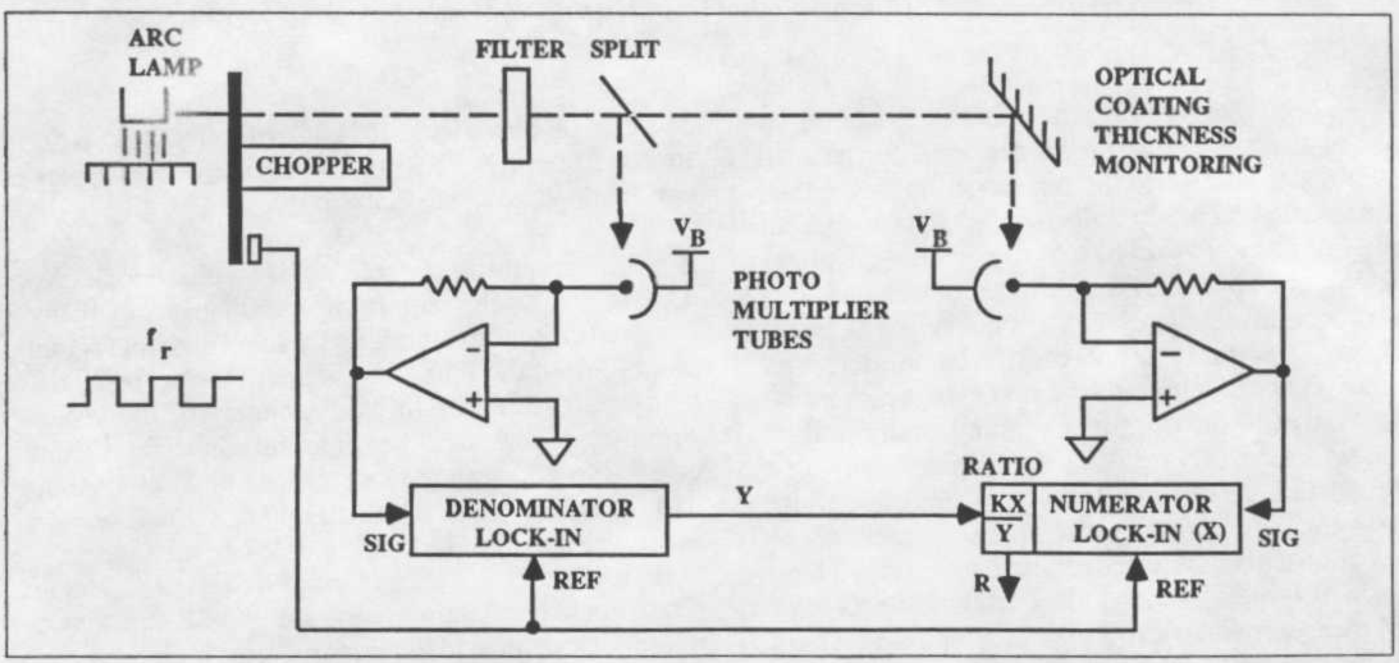


FIGURE 5 DUAL DETECTOR, SINGLE FREQUENCY, DUAL LOCK–IN RATIOMETRY

III DUAL LOCK-IN TECHNIQUES

Now we are progressing to some real performance enhancements. First of all, the setup shown in Figure 5 gets rid of the background radiation and ambient light offsets in the denominator path. Furthermore, the use of a second lock–in suppresses dc bias effects such as photomultiplier dark current or voltage biasing of photoconductive detectors. One can also reduce potential source fluctuation problems by an order of magnitude or two by setting both lock–ins to the same time constant to balance the X and Y transfer functions. The accuracy level achievable with two lock–ins justifies the use of digitially calculated ratios in preference to analog circuit methods. Note that the diagram shows the ratiometer where it typically resides physically, inside the numerator lock–in. Many microprocessor based lock–ins, such as the ITHACO Models 3961 and 3962 provide this function digitally with an external A/D input for Y. If the source variations are relatively small, the denominator lock–in need not be very fancy, even for very high accuracy work. A low cost Model 3921 will be more than adequate.

In Figure 6, a Model 220 Chopper with a dual row blade chops separate X and Y beam paths at two frequencies. The beams are recombined onto a single detector and the signals then are separated by two lock–ins synchronized to the two chopping frequencies. This technique has all of the advantages listed for the dual detector setup shown in Figure 4. In addition, the use of only one detector eliminates the drift and responsivity mismatch problems associated with having separate X and Y detectors. Note that the lock–ins block the 50 Vdc detector bias while responding to signals of perhaps 50 microvolts rms.

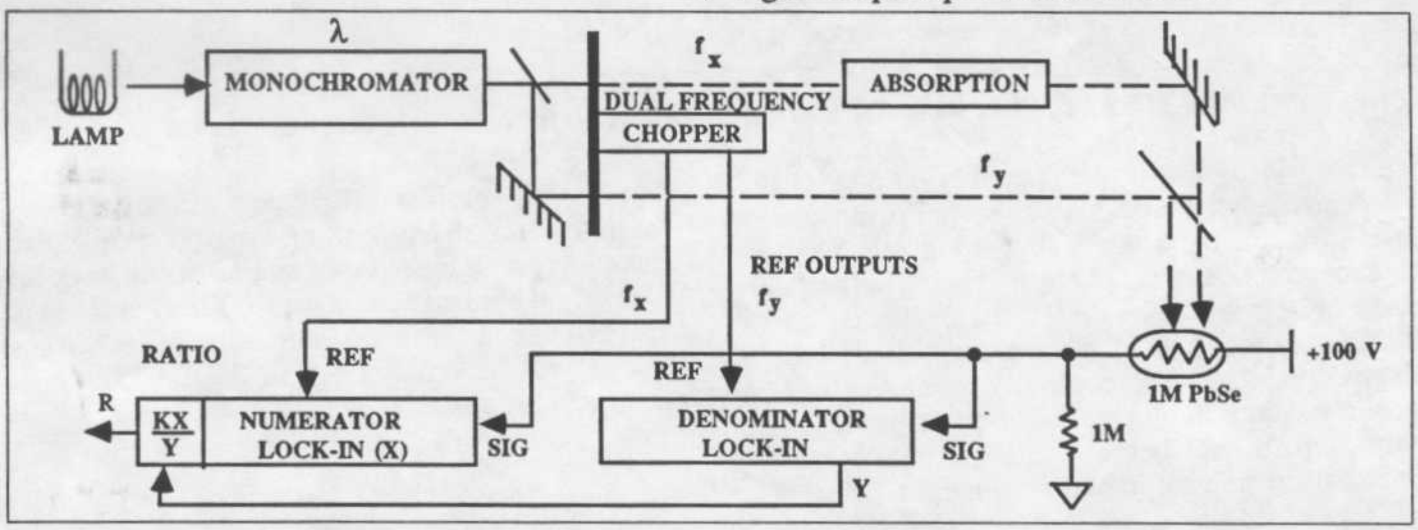


FIGURE 6 DUAL BEAM, DUAL FREQUENCY, DUAL LOCK–IN, SINGLE DETECTOR RATIOMETRY

IV SINGLE PHASE AND TWO PHASE LOCK–INS

In fixed detector, fixed beam geometry systems, the phase of the numerator or the denominator lock–in signal inputs relative to the reference input will tend to remain constant. In this sort of electro–optical work, one may very conveniently use single phase lock–ins since phase adjustments are a "set–and–forget" procedure. If the optical geometry is not fixed, but instead aperture sizes are changed or the detectors are moved about within the beam or the beam translates relative to the original chopping position, then phase will tend to vary. The signal phase may also shift with increasing modulation frequency due either to detector lag or preamplifier phase shift. For these applications, one is advised either to use a dual–phase lock–in for phase independent magnitude (vector sum) measurement, or to use a single phase lock–in with Autophase capability. The ITHACO Model 3921 single phase lock–in is of particular interest due to its smoothly tracking rather than reset–upon–command Autophase feature.

A word of warning concerning dual–phase lock–ins: The analog vector summing circuit used in many designs limits you to a relatively small absolute signal range. When the numerator signal falls 10 to 20 dB below full scale, the limited accuracy of the vector summing process will begin to introduce significant errors. For wide range work (X and or Y change 60 dB) in which it is not desirable to switch lock–in sensitivity (to avoid glitching or range–change calibration errors ~ 0.5%), one is advised to use a single phase lock–in. Alternatively, one could use a two phase lock–in provided it incorporates high accuracy (e.g., 16–bit) digital computation of vector summation, such as provided by the ITHACO Model 385EO Integrator/Coupler. Observe that the limitations of analog vector summers applies regardless of the accuracy or method of ratio calculation, digital or analog, or of the denominator derivation technique, dc or ac.

Another hazard with vector sum lock–ins is the rectification of noise, as described in IPB 0121. This causes a non–ratiometric additive error in both numerator and denominator channels which can be suppressed only at the expense of choosing a sufficiently long time constant or integration averaging time ahead of the vector summing process.

V DYNAMIC RATIO PHASE CONSIDERATIONS

Random changes in the signal source will in general have differing in–phase and quadrature components. If the source fluctuates rapidly relative to the modulation (e.g. chopping) frequency this can have a disturbing effect on dynamic ratiometric cancellation of source effects, owing to the phase sensitive nature of lock–in amplifiers. To understand this consider the case of a signal source that switches randomly in time from 0 to maximum output and back to zero. (This corresponds to rapid laser mode jumping.)

In Figure 7 the 0° and 90° reference waveforms represent two lock–ins phased far apart (large error condition). The 0° and 90° phase sensitive detector (PSD) signals correspond to the respective outputs of the two lock–ins prior to time constant lowpass filtering. The lack of phase correlation between these two outputs would lead to a failure of dynamic ratio cancellation if they were used as numerator and denominator signals.

The first conclusion to be drawn by inspection of Figure 7 is that, in dual lock–in setups, both instruments must impart the same phase shift to the reference relative to the signal if the source fluctuates rapidly relative to the reference frequency. This might happen, for example, when using an arc lamp source. This would be true both for single phase and dual phase lock–ins. Also one cannot mix single phase and vector sum lock–ins in the same setup. Furthermore, one cannot suppress these very rapid fluctuations in a dual frequency setup as in Figure 6 due to noise incoherence at the two measurement frequencies.

The second conclusion is that a single lock–in (dc denominator) setup can never work for such rapid source fluctuations. To see this, observe that the partial correlation between the signal (Y) waveform in Figure 7 and either the 0° or 90° (X) waveform will lead to imperfect cancellation of fluctuations. Attempts to circumvent this effect by using a two phase lock–in will be limited by the noise rectification property of the vector sum computation.

The third conclusion is that if the source fluctuates slowly or infrequently relative to the reference frequency, then dynamic ratiometry will work without errors due to phase mismatch. Accuracy will improve approximately linearly as a function of the degree to which the reference frequency exceeds the fluctuation rate. In this case, both dual lock–in and dc denominator techniques will yield comparable dynamic performance, and single phase lock–ins can be mixed with dual phase units.

Observe that for pulsed lasers which exhibit jitter in pulse position or pulse width, only a dual lock–in ratio technique will suffice to suppress the jitter, and that both lock–ins must be phased. Pulse height jitter, on the other hand, does not require phased lock–ins, and will function in dc denominator setups as well.

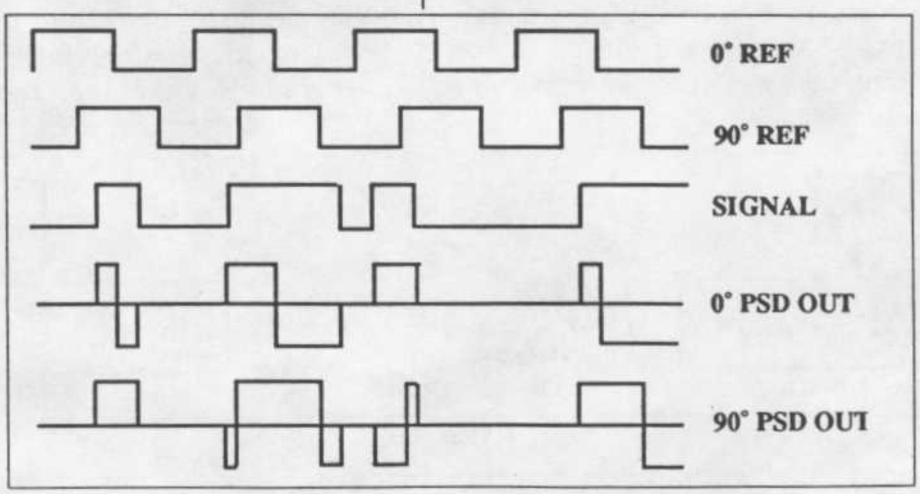


FIGURE 7 STEP FUNCTION PHASE RESPONSE

VI DIGITAL TECHNIQUES – LIMITATIONS OF LOW COST APPROACHES

VI-A TIME SIMULTANIETY CONSIDERATIONS

As part of the balanced transfer function requirement described in Appendix A for accurate transient ratio work, it is assumed there is no time skew in the determination of X and Y. For purely analog ratiometers, this presents no problems since the circuits work in real time. For digitally computed ratios one has to be more careful. In most instances, digital ratiometry involves a multiplexed ADC which samples the X and Y signals sequentially. This can destroy the transient performance if the samples are not taken rapidly compared to the lock–in time constant or source fluctuation slew rate (whichever is slower).

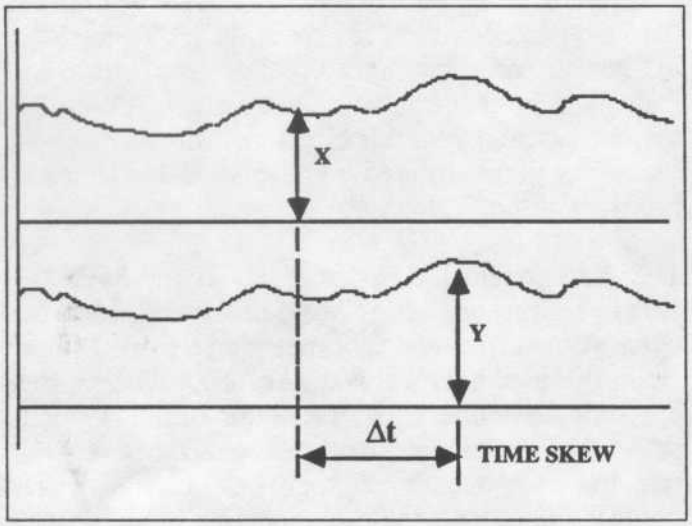


FIGURE 8 TIME SKEWED SAMPLING

VI-B AVERAGING TO OVERCOME TIME SKEW

One technique for partially overcoming sampling simultaneity problems is the use of signal averaging. In this approach the lock–in time constant is set short and a large number of rapidly acquired digital samples are averaged for the X and Y paths. By this means the lock–in time constant setting becomes insignificant in the smoothing of X and Y data and the length of time t_m over which samples are averaged becomes dominant in the smoothing process. Assuming N samples are taken at a rate at least twice as fast as the lock–in bandwidth ($B_L = 1/(8T)$ at 12 dB/octave rolloff), the overall bandwidth B_A of the averaging process becomes $1/(2t_m)$. For example, using a Model 3961 or 3962 in the linear averaging mode with a 30 msec time constant T, a 100 msec sampling interval t_s and 128 sampling averaging we have:

$$
\begin{aligned}
B_L &= 1/(8T) &&= 4.17 \text{ Hz (12 dB/oct)} \\
r_s &= 1/t_s &&= \text{sampling rate} = 10 \text{ Hz} \\
t_m &= N\,t_s &&= 12.8 \text{ sec} \\
B_A &= 1/(2t_m) &&= .039 \text{ Hz}
\end{aligned}
$$

The value of B_A yields a smoothing equivalent to a lock–in time constant equal to $(1/4)\,t_m = 3.2$ seconds.

One would employ this approach when using a pair of 3961 and 3962 lock–ins in a dual lock–in setup in which time a simultaneity between lock–ins of 100 msec, in conjunction with the desired value of t_m, gives adequate suppression of source transients (e.g., about 120:1 for the conditions given above). The averaging technique overcomes the fact that these lock–ins do not have a trigger input to synchronize data acquisition.

Both lock–ins are set to identical values for number of samples, sampling rate, time constant, and rolloff dB/octave. The lock–ins are both phased for a maximum Acosø output. The Acosø DAC output of the denominator lock–in (not PSD output, it is not averaged) is connected to the EXT-IN input of the numerator lock–in and the ratio mode of the latter is selected. To begin, one sets both lock–ins to the linear averaging mode. The ratio becomes valid t_m seconds later and remains valid thereafter. This latter characteristic is a consequence of the running average maintained by both lock–ins, in which the eldest data is discarded after N samples have been accumulated.

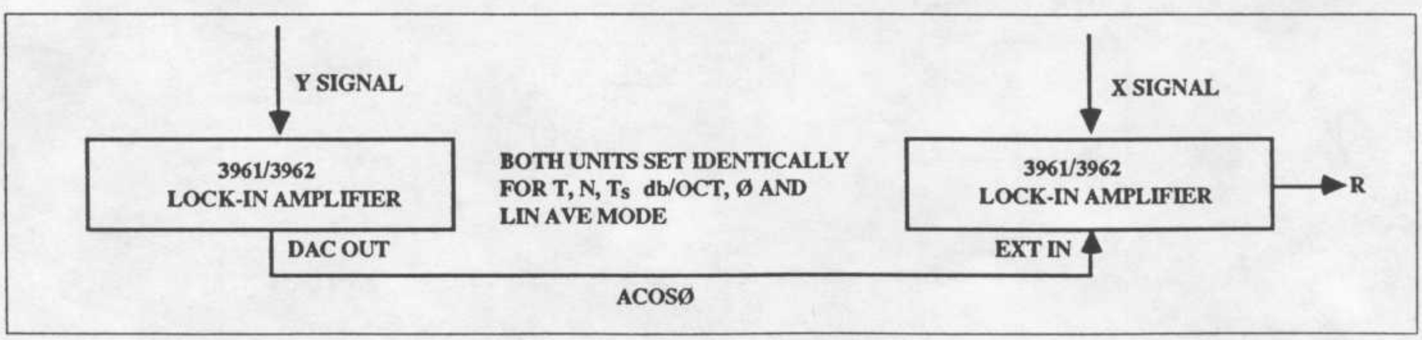


FIGURE 9 USING 3961/3962 LOCK–INS IN SIGNAL AVERAGING MODE TRANSIENT RATIOMETRY

VI-C ADC RESOLUTION

For wide dynamic range work, limited bit resolution will result in limited accuracy. For example, the 3961/3962 Lock–Ins are good to 10 bit resolution full scale in the worst case (1 part per 1000). Thus if the signal fell 20 dB below full scale for either the X or Y dc input, the resolution would be only 10% of 10 bits or 1% of reading.

To overcome this problem, the X and Y lock–in gain would have to be switched at 10 dB (3:1) intervals in a dual lock–in setup, which would yield a worst case 0.3% of reading resolution error. In a single lock–in setup, the Y input would be similarly limited to the 3 to 10 V dc range for 0.3% resolution unless gain switching were provided in that path.

Manual gain switching is inconvenient. Doing it from a host computer is best, as the gain scaling can be compensated for by software. Autoranging within the 3961 or 3962 is a problem, since the ratiometer works on the dc 0–10 volt output of the lock–in and does not compensate for sensitivity changes. You cannot readily keep track of where you are and compensate the K factor when the gain changes. Any gain switching within the lock–in also introduces glitches in the ratio reading which will require a settling time dependent on the time constant T and/or the averaging time $t_m = N t_s$.

VII THE 385EO INTEGRATOR/COUPLER AS DIGITAL RATIOMETER

The above discussion points out the limitations of low cost digital ratiometry involving time skewed sampling, low bit resolution, Autoranging within the lock–in and lack of gain scaling provisions. The performance is not much better than analog techniques can provide, and is some respects the results are inferior, such as for transient ratiometry.

The Model 385EO Integrator/Coupler solves all of these problems in an elegant and extremely accurate way. This unit incorporates two AUTO–GAIN®, bipolar, A to D converters which are triggerable for time simultaneous sampling and which provide up to 16 bit resolution (20 bit dynamic range with AUTO–GAIN enabled). The ADC circuits operate on a voltage–to–frequency principle and thus provide averaging (integration over a selectable time period t_m ranging from 1/60 second to 2.78 minutes) similar to that discussed in Section VI-B. This unit also provides digital baseline correction to null residual input offsets. These features result in near ideal performance to unlock the full ratiometric potential of lock–in amplifiers.

1. Simultaneous triggerability for both channels eliminates time skew and allows external synchronization with other system events.

2. By using an integration time t_m much larger than the lock–in time constant $\bar{T}$, the 385EO can be made to dominate the filtering transfer functions for the X and Y paths. The transfer function accuracy of the 385EO is determined by its internal timing accuracy (about 0.01% worst case for t_m =1 sec) and is independent of electrical component values. This filtering match applies both for dual lock–in and dc denominator setups.

   Note that time constant component errors within the lock–ins become insignificant in dynamic dual lock–in applications when using the385EO as the ratiometer with t_m >>4T. Also note that the external RC filter as shown in Figure 3 in dc denominator setups, is eliminated.

3. AUTO-GAIN within the 385EO over a 1:10:100 range eliminates glitching. Furthermore, the 385EO automatically keeps track of and compensates for scaling changes without manual or host computer intervention. Extremely wide dynamic ranges (1% accuracy at 60 dB below full scale) can be accommodated without changing any lock–in parameter or control.

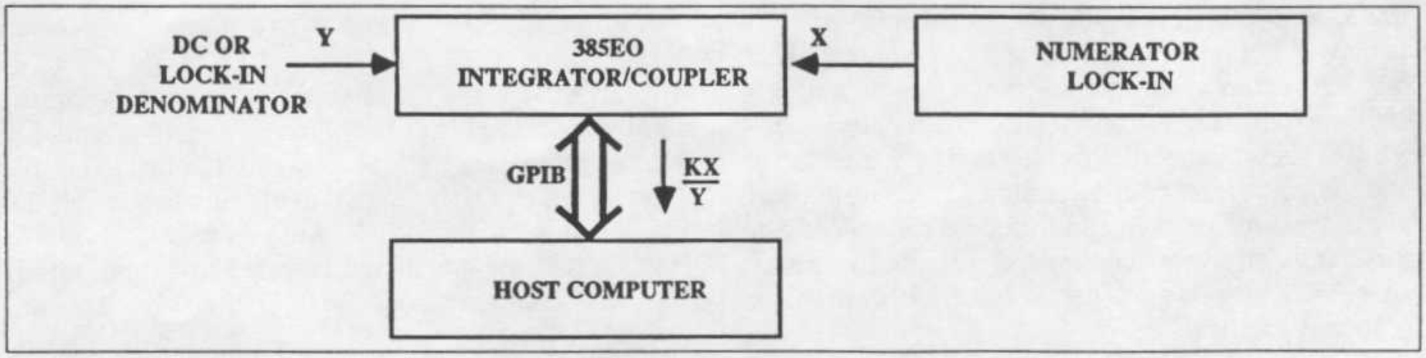


FIGURE 10 THE 385EO AS RATIOMETER

4. Changing the integration time t_m provides glitchless equivalent time constant switching as the signals fall toward the system noise floor. Thus a cleanly performing, adaptive, filtering function can be incorporated under host software control.

5. Digital baseline correction allows baseline nulling to 60μV on the 10V scale, which is commensurate with the 10 ppm (100 V) output zero stability of top–of–the line DYNATRAC Lock–Ins such as the ITHACO Models 391A, 393 and 399. Its this capacity that makes the 60 dB dynamic range mentioned in #3 above a practical reality.

6. ADC linearity over a 100:1 range is typically 0.1% of reading or better. This wide range accuracy is due to AUTO–GAIN, which circumvents the 1% to 0.5% gain switching accuracy limitations of lock–in amplifiers, and due to the extended linearity inherent in V–F converters . Over smaller ranges (e.g., X and Y vary 2:1) 0.01% accuracy can be expected.

7. Ideally, a single 385EO would serve two lock–ins operated single phase (use Acosø rather than vector sum outputs.) In an application demanding two vector sum lock–ins and either very high accuracy or operation over a very wide dynamic range, one could employ two 385EO units. Each would compute a vector sum and report the value to the host computer, which would then calculate R = KX/Y. Time simultaneity can be maintained by triggering both 385EO units using the GPIB Group Trigger command.

8. Data rates up to 60 samples/second are possible at the10 bit resolution level.

VIII REPRESENTATIVE SYSTEMS

VIII-A ROCK BOTTOM LOW PRICE, 3921 WITH RATIO OPTION 02

This simple analog ratio, dc denominator instrumentation will implement Figures 1, 2 and 4 in the 10 Hz to 15 kHz reference frequency range. It will handle a 40 dB denominator range with approximately 2% linearity. This system will provide good performance in moderate interference environments where its typical 40 dB dynamic reserve and limited 6 dB rolloff can sufficiently suppress random and discrete frequency noise. 3921 lock–ins will perform accurately in swept frequency applications.

VIII-B DUAL 3921, ONE WITH RATIO OPTION 02

With two lock–ins much improved performance is possible allowing economical implementation of Figures 5 and 6 for the suppression of background radiation and electrical dc offsets. Autophase tracking of the reference in both lock–ins allows for fairly large range transient ratiometry (20 – 30 dB fluctuations) with good accuracy and the suppression of very rapid source fluctuations per Section V.

VIII-C 3925 SYSTEM; TWO 3921 LOCK–INS PLUS 385EO AS RATIOMETER

This yields excellent performance. Autophasing in conjunction with the digitizing and filtering precision of the 385EO Integrator/Coupler provides both static and transient accuracy unobtainable in many higher priced competitive systems. It is recommended wherever a low priced, computer interfaced setup is desired.

VIII-D 3961 OR 3962 ALONE

These lock–ins provide simple, digital, completely programmable dc denominator ratiometry as a standard feature to implement Figures 1, 2 and 4. Compared to the 3921 they offer a much better ability to cope with high levels of interference due to a dynamic reserve of up to 100 dB and due to 12 dB/octave time constant rolloff. They also cover a much wider frequency range, from 0.5 Hz to 200 kHz. One disadvantage involves non–simultaniety of sampling (ability to perform transient ratiometry is limited to cases in which the source fluctuation is much slower than the reference frequency but much faster than the time constant, and an external, matched, denominator RC filter must be inserted). Other disadvantages of the 3961 or 3962 alone include 10 bit resolution, which limits dynamic range, and inability to use AUTO–GAIN effectively. The use of a tracking bandpass filter to suppress harmonic responses places a severe limit on the ability of these lock–ins to operate in swept frequency applications.

VIII-E DUAL 3961 OR 3962 SYSTEM

The advantages and disadvantages for this system are as above, except that background radiation and dc offset problems are eliminated and that limited transient ratiometry using signal averaging is possible as explained in Section VI–B. Autophase in both lock–ins is a definite advantage in cases involving rapid source fluctuations. As in VIII–D, the slow data rate

(6 samples/sec for RS-232, 2 second for GPIB) could be a problem. For lower cost, a 3921 could replace the denominator lock–in (with loss of transient matching). For greater accuracy, higher speed and the ability to handle more demanding transient requirements, one can use the 385EO to digitize the Autophased Acosø dc outputs of both lock–ins.

VIII-F DYNATRAC ANALOG RATIOMETRY

The top–of–the–line DYNATRAC® Heterodyning Lock–Ins, Models 391A, 393 and 399, also can be provided with Analog Ratio Option 02. Compared to the 3921, they provide a much higher capacity for dealing with interference due to dynamic reserve greater than 86 dB and due to 12 dB/octave time constant rolloff. DYNATRAC instruments provide greater accuracy, dynamic range and baseline stability than the 3961/3962 units and work much better with very low level or very high frequency signals. Heterodyning allows the rapid tracking of signal frequency in swept frequency measurements. In chopper modulated work, it accommodates a fluctuating reference input frequency without phase errors, whereas the tuned signal conditioning filter of the 3961/3962 could lead to real headaches. Disadvantages of DYNATRAC systems include lack of autophasing and the need to change cardsets to cover the entire 0.1 Hz to 200 kHz frequency range.

The 397EO Lock–In covers from 10 Hz to 10 kHz and has only a 48 dB dynamic reserve. Ratiometry can be accomplished only by using its analog vector sum, which limits its accuracy and dynamic range.

DYNATRAC analog ratiometry can incorporate any dc technique or lock–in as the denominator source. For transient work, an identical DYNATRAC is recommended.

VIII-G DYNATRAC DIGITAL RATIOMETRY

Use of the 385EO to digitize the dc outputs of DYNATRAC Lock–Ins, Models 391A, 393 and 399, yields the full accuracy potential of ratiometric techniques, as described in Section VII. The results are unsurpassed by any competitive equipment.

The use of the 397EO with the 385EO is limited due to its lack of a phasing control. One cannot obtain single phase Acosø performance unless an external phase shifter is inserted in the reference input path.

For dual lock–in DYNATRAC digital ratiometry employing the 385EO Integrator/Coupler, a very inexpensive lock–in, (e.g., Model 3921) can be used in the denominator path without sacrificing transient performance or accuracy. This will be true if the lock–in time constants are very short compared to the integration time and that the denominator fluctuations are relatively shallow (e.g., 2:1).

APPENDIX A

TRANSIENT RATIO ERROR DUE TO UNBALANCED TIME CONSTANTS

By Hans G. Jorgensen
Eng. Report #95032, 2 August 1977

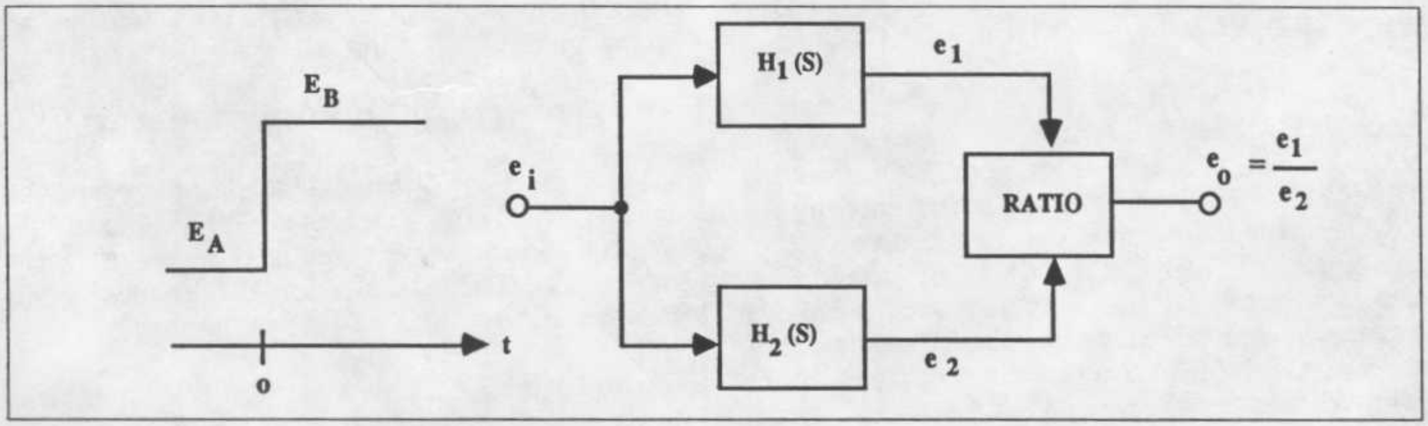


FIGURE 11 TRANSIENT ANALYSIS BLOCK DIAGRAM

The purpose of this report is to compute the maximum error in the ratio between the outputs of two lock–ins (e_1 and e_2) if the time constants are not balanced. Results are shown for the 6 and the 12 dB per octave cases with a voltage step from E_A to E_B (at time $t = 0$).

For the 6 dB per octave case the transfer functions are:

$$H_1(S) = \frac{1}{1 + T_1 S} \quad \text{and} \quad H_2(S) = \frac{1}{1 + T_2 S}$$

We obtain for $t \geq 0$:

$$e_o = \frac{1 + (E_A/E_B - 1)\ \exp(-t/T_1)}{1 + (E_A/E_B - 1)\ \exp(-t/T_2)}$$

For the 12 dB per octave case we assume identical time constants in each lock–in. This is probably the worse case but no proof is given. The transfer functions are:

$$H_1(S) = \frac{1}{(1 + T_1 S)^2}$$

$$H_2(S) = \frac{1}{(1 + T_2 S)^2}$$

for $t \geq 0$ we obtain:

$$e_o = \frac{1 + (E_A/E_B - 1)\ [\exp(-t/T_1) + (t/T_1)\exp(-t/T_1)]}{1 + (E_A/E_B - 1)\ [\exp(-t/T_2) + (t/T_2)\exp(-t/T_2)]}$$

The maximum ratio error was found in each case by iteration using a programmable calculator. The time constants were assumed to be ***unbalanced by 1%:*** $T_1 = 1$; $T_2 = 1.01$

Peak ratio error for 1% unbalance:

E_A/E_B	6dB/Oct	12dB/oct
20	–1.50%	–2.00%
.05	+.71%	+1.18%

Note that if both lock–ins have time constants determined by 1% tolerance resistors and capacitors the maximum ratio error in the worst (but very unlikely) case is 8%.

APPENDIX B

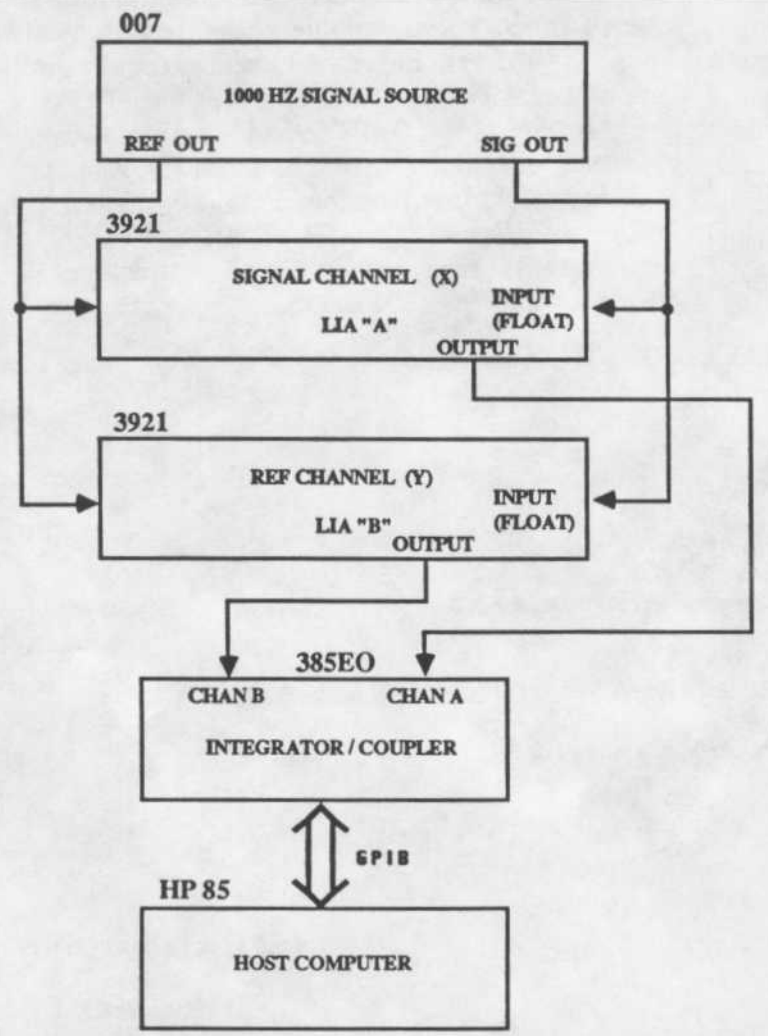
DEMONSTRATION EXAMPLE

3925 SYSTEM CANCELS RAPID SOURCE NOISE

The experiment described below illustrates the near ideal ratiometric performance obtainable by operating two 3921 Lock–In Amplifiers into a single 385EO Integrator/Coupler acting as ratiometer. The setup, as pictured in Figure 12, simulates the dual detector, single frequency arrangement shown in Figure 5. The special ITHACO–manufactured signal source called the Model 007 generates a 1 kHz reference, 1 kHz signal and approximately a dc to 1 kHz flat spectrum random noise interference. The Model 007 settings as shown produce a sinewave signal $e_s = 10.0$ mv rms mixed with a broadband noise of roughly 100 mV peak to peak (density, $e_n = 0.5\ mV/\sqrt{Hz}$ rms.)

Thus, the setup demonstrates the dynamic suppression of an extremely heavy source noise. The speed and accuracy improvement over brute force time constant filtering is dramatic. The great simplicity with which this is done is due to the Autophase tracking capability of the 3921 Lock–In and to the superior data processing characteristics of the 385EO Integrator/Coupler.

To quantify the performance improvement, the HP-85 applications program "25DEMO" as described in IAN 42, "Ratiometric and Noise Measurement Demonstration Programs for Model 3921 Lock–In Amplifier with Model 385 Integrator/Coupler", was modified to calculate the standard deviation of repeated ratiometric measurements. This program normally provides a signal reading scaled in percent relative to an initial reading. The changed version also calculates the $\pm 1\sigma$% deviation from the mean for X, Y and R.



007 SETTINGS

ALL OFF EXCEPT:

NOISE = 5.00
1 kHz = 1.00 (10 mV)
PHASE = 0°

$e_s = 10$ mV

$e_n \cong 0.5\ mV/\sqrt{Hz}$

LIA SETUP

	"A" & "B"
TIME CONST	.01 sec
SENS	10 mV
MULT	3.00
INPUT	FLOAT
ZERO OFFSET	0.00
POST FILTER	OFF
0°/90°	0° *
0°/180°	0° *
AUTO/MAN	AUTO
LAMP	ON

* or as required if phase of 007 is not set to 0°

FIGURE 12 DEMONSTRATION SETUP USING 007 SIGNAL SOURCE WITH MODEL 3925 RATIOMETRIC SYSTEM

The controls of both lock–ins were set identically the same, with TIME CONSTANTS set to the minimum value of T = 10 milliseconds. The 385EO integration time was set to $t_o = 2$ seconds to dominate system bandwidth B and thus to obtain an exactly balanced filtering function. To null residual dc offsets and thereby achieve maximum accuracy, the 385EO baseline correction was invoked first with both lock–in inputs shorted. A signal level of 10 mV was then applied to produce a 1/3 of full scale reading (f.s. = 10 mV x 3.00 = 30 mV rms using SENSITIVITY and MULT controls) with noise off, and the K factor was obtained using the NORM function of the program. This signal level leaves ample headroom for operation without overload due to output fluctuations when noise is turned on. The expected signal measurement fluctuation as viewed at the output of the lock–ins is $E_n = e_n \sqrt{B} = e_n \sqrt{1/4T}$ = 2.5 mV rms (±7.5 mV peak or ±75% of reading), as explained in IAN 49 "Speed/Accuracy Tradeoff When Using a Lock–In to Measure Signal in the Presence of Random Noise".

With noise turned on, the dc outputs did indeed exhibit the large fluctuations as predicted above. As integrated by the 385EO, these fluctuations were attenuated to ±2.32% rms of reading reproducibility error for the X and Y outputs, as measured by calculating the standard deviation of 1200 readings. This is in good agreement with approximate expected value of $(100\% \times e_n/e_s)\sqrt{1/(2t_o)} = \pm 2.5\%$ rms for the integrator alone.

In contrast, the ratio measurement deviation was only ±0.054% at the same ±1σ level, a factor of 44 times better than obtained by signal averaging in the X or Y channels! To obtain this high accuracy using only 385EO integration would require $t_m = (1/2)\,[e_n/(e\,\sigma_x)]^2 = 1/2\,[0.5/(10 \times .00054)]^2 = 4300$ seconds = 1.2 hours. Thus we could also view the dynamic, ratiometric improvement in speed/accuracy tradeoff as a 2000 fold decrease in measurement time to achieve the same 0.054% reproducibility.

The 3925 also can dynamically cancel source discrete frequency interference, such as 60 Hz harmonics. When the experiment above was repeated with noise off and 10 mV of 990 Hz mixed with the 10 mV of 1 kHz signal, the large 10 Hz beat frequency in the lock–in output was suppressed to 0.034% in 2 seconds, whereas the X and Y data exhibited ±0.17% reproducibility error. This corresponds to a 5X accuracy improvement or a 25X speed improvement over signal averaging or long time constant smoothing.

These results show the excellent results achievable using the Model 3925 Ratio System. The described technique is directly applicable to situations involving laser pulse jitter or arc lamp noise. Note that the 3921 Lock–In operates accurately and continuously in swept frequency and variable phase measurements. Thus the 3925 system is hard to beat except in applications where its limited frequency response (15 kHz) or dynamic range (~50 dB) demands the use of a more elaborate lock–in in one or both channels. And other lock–ins would involve other compromises, such as lack of autophase (DYNATRAC 391A, 393, 399) or discrete rather than continuous phase tracking and frequency tracking (3961, 3962).

DL INSTRUMENTS, LLC, 233 C ecil A. Malone Dr., Ithaca, New York 14850; Phone 607-277-8498; Fax 607-277-8499